Int. J. Solids Structures Vol. 22, No. 11, pp. 1369–1373, 1986
Printed in Great Britain

0020-7683/86 $3.00 + .00
Pergamon Journals Ltd.

DISCUSSION OF

David T. Barr and Michael P. Cleary, "Thermoelastic fracture solutions using distributions of singular influence functions—II. Numerical modelling of thermally self-driven cracks", *Int. J. Solids Structures*, Vol. 19, No. 1, pp. 83–91 (1983)

For possible application to the "Hot Dry Rock" geothermal system, the authors[1] consider an infinite array of equally spaced parallel (planar) cracks of *common initial length, growing at a common rate* because of the cooling induced by heat extraction (a two-dimensional problem). On the basis of this model, they seem to suggest that initially equal parallel cracks can continue to grow at a common rate (i.e. remain of equal length), once their common length exceeds twice their common spacing.

I wish to point out that a more complete analysis which would permit the *possibility* of *unequal* crack growth would immediately reveal that parallel cracks driven by thermal cooling *do not remain of equal length*, once their common length attains a critical value (measured in terms of crack spacing as the unit length). Therefore, the only reason the authors do not predict unequal crack growth in their calculations is that they *a priori* preclude this possibility by *assuming* that all cracks remain of the same length throughout the entire growth process.

As has been shown both theoretically and experimentally by this writer and co-workers[2–5], the growth pattern of parallel cracks driven in linearly elastic brittle solids by convective or conductive (or a combination of both) cooling, inevitably involves unequal crack lengths, unless one precludes this possibility in the analysis by allowing for only one common crack length, as do the authors. This fact can be easily established for the most general temperature field that may be generated by convective, conductive, or a combination of both, heat extraction processes.

To show this, consider an infinite array of parallel edge cracks, and allow for the *possibility* of unequal crack growth, in the manner sketched in Fig. 1: every other crack is of length L_1, the others are of length L_2; the case of equal cracks then is the special one with $L_1 = L_2 = L$. Let at an instant the temperature *field* be denoted by $\theta = \theta(x, y)$, with appropriate symmetry corresponding to Fig. 1.

In view of symmetry, only the opening mode stress intensity factor controls the fracture process. Let $K_i = K_i(L_1, L_2; \theta(x, y))$, $i = 1, 2$, be the stress intensity factor at the tip of the ith crack, and observe that K_i is a *functional* of the temperature *field* $\theta(x, y)$.

For both cracks to continue to be active, it is *necessary* that

$$K_1 = K_c \quad \text{and} \quad K_2 = K_c \tag{1}$$

where K_c is the critical value of the stress intensity factor. The solution of eqns (1) gives, as the temperature field θ is varied, the common (equilibrium) crack length, $L_1 = L_2 = L$. Since, at a fixed temperature *field*, we have, in general

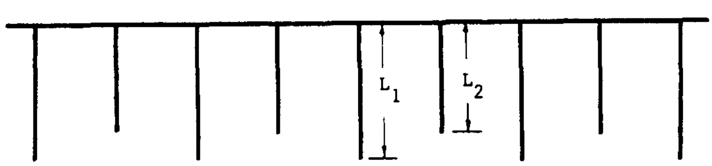

Fig. 1.

$$\frac{\partial K_1}{\partial L_1} = \frac{\partial K_2}{\partial L_2} < 0 \qquad \text{for} \quad L_1 = L_2 \tag{2}$$

the crack growth is *stable*, in the sense that for an incremental growth, an increment of heat must be removed.

This, however, does not mean that the equal crack growth pattern is energetically preferred no matter what the common crack length L may be.

In fact, since the equilibrium conditions (1) must be maintained for continued crack growth induced by variations $\delta\theta(x, y)$ in the temperature field, we must have (because K_c is assumed constant)

$$\begin{aligned} \frac{\partial K_1}{\partial L_1}\mathrm{d}L_1 + \frac{\partial K_1}{\partial L_2}\mathrm{d}L_2 + \delta_\theta K_1 = 0 \\ \frac{\partial K_2}{\partial L_1}\mathrm{d}L_1 + \frac{\partial K_2}{\partial L_2}\mathrm{d}L_2 + \delta_\theta K_2 = 0 \end{aligned} \tag{3}$$

where $\delta_\theta K_i > 0$ is the change (in the functional sense) in K_i produced by the variation (because of the heat removal) in the temperature *field*, $\delta\theta(x, y)$, at constant L_1 and L_2. Furthermore, for strain-controlled problems of this kind, we always have

$$\frac{\partial K_i}{\partial L_j} \leqslant 0 \quad \text{at}\ \theta(x, y)\ \text{fixed}, \qquad i, j = 1, 2. \tag{4}$$

At given equilibrium states with $K_1 = K_2 = K_c$ and $L_1 = L_2 = L$, eqns (3) yield the unique solution $\mathrm{d}L_1 = \mathrm{d}L_2 = \mathrm{d}L$, as long as the determinant of the coefficients of $\mathrm{d}L_1$ and $\mathrm{d}L_2$ is positive. However, once the condition

$$\left(\frac{\partial K_1}{\partial L_1}\right)\left(\frac{\partial K_2}{\partial L_2}\right) - \left(\frac{\partial K_1}{\partial L_2}\right)\left(\frac{\partial K_2}{\partial L_1}\right) \leqslant 0 \tag{5}$$

is satisfied, then another solution with, say, $\mathrm{d}L_2 = 0$ and $\mathrm{d}L_1 > 0$, becomes possible. It is easy to show, see Nemat-Nasser *et al.*[3], that this new solution also involves *stable* crack growth, i.e. to increase L_1 at constant L_2, an increment of energy must be removed and, hence $\delta_\theta K_i > 0$. However, if condition (5) is satisfied, then the solution $\mathrm{d}L_2 = 0$, $\mathrm{d}L_1 > 0$, is *energetically preferred*, because it leads to a smaller stored elastic energy than the solution with $\mathrm{d}L_1 = \mathrm{d}L_2 \neq 0$. These facts have been thoroughly discussed and illustrated by this writer and co-workers in a series of papers, see, e.g. Refs [2–5]. They are also supported by experimental observations. An example taken from Geyer and Nemat-Nasser[4] is shown in Fig. 2.

Indeed, thermal cracks of the kind discussed by the authors are very much similar to the shrinkage cracks observed in wood, in concrete, or in dried-up clay deposits. One invariably observes a nested sequence of cracks, which involves many shallow closely-spaced cracks of more or less hexagonal pattern, encompassed by deeper and further apart cracks, leading to large, deep cracks of a similar pattern. The example of Fig. 2 is a two-dimensional nested crack pattern of this kind.

It therefore seems that, while the results presented by the authors are interesting, they only apply to the very beginning of the process of thermally induced crack growth, where the common crack length is rather small relative to the common crack spacing, and not

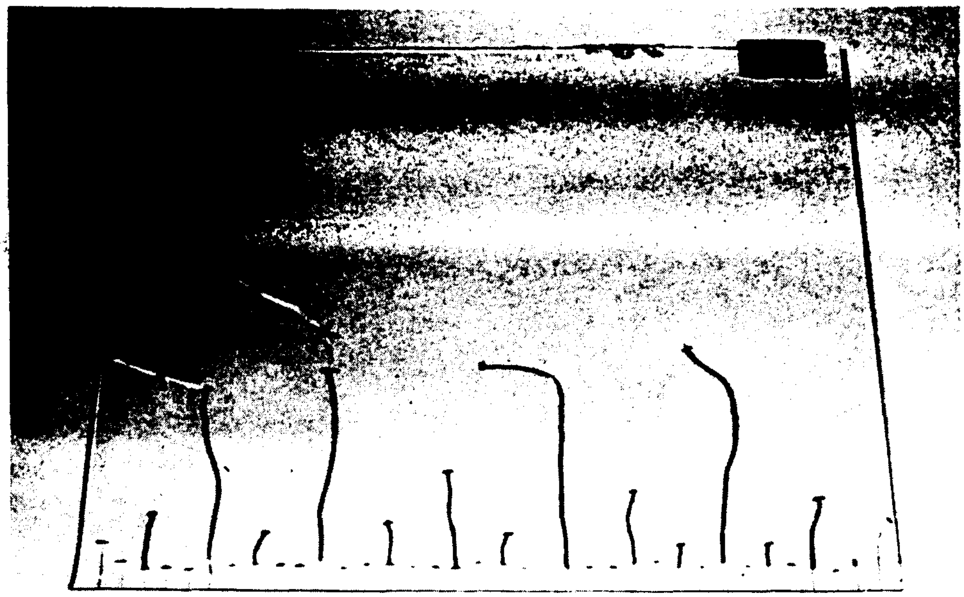

Fig. 2. Crack regime for plate with initial cracks; crack ends are marked at the end of the experiment (from Geyer and Nemat-Nasser[4]).

to the case where the common crack length is large relative to the spacing, contrary to what is suggested by the authors.

S. NEMAT-NASSER
Department of Applied Mechanics and Structural Engineering
University of California
San Diego, La Jolla, CA 92093
U.S.A.

REFERENCES

1. D. T. Barr and M. P. Cleary, Thermoelastic fracture solutions using distributions of singular influence functions—II. Numerical modelling of thermally self-driven cracks. *Int. J. Solids Structures* **19**, 83–91 (1983).
2. S. Nemat-Nasser, L. M. Keer and K. S. Parihar, Unstable growth of thermally induced interacting cracks in brittle solids. *Int. J. Solids Structures* **14**, 409–430 (1978).
3. S. Nemat-Nasser, Y. Sumi and L. M. Keer, Unstable growth of tension cracks in brittle solids: stable and unstable bifurcations, snap-through, and imperfection sensitivity. *Int. J. Solids Structures* **16**, 1017–1035 (1980).
4. J. F. Geyer and S. Nemat-Nasser, Experimental investigation of thermally induced interacting cracks in brittle solids. *Int. J. Solids Structures* **18**, 349–356 (1982).
5. S. Nemat-Nasser, Thermally induced cracks and heat extraction from hot dry rocks, *Hydraulic Fracturing and Geothermal Energy* (Edited by S. Nemat-Nasser, H. Abé and S. Hirakawa), pp. 11–31. Martinus Nijhoff, The Hague (1983).

Int. J. Solids Structures Vol. 22, No. 11, pp. 1373–1375, 1986
Printed in Great Britain.
0020–7683/86 $3.00 + .00
Pergamon Journals Ltd.

AUTHORS' RESPONSE

The stability arguments presented in the above Discussion do not apply universally to all crack arrays in all situations. As an extreme case, for which the stated crack behaviors are not obtained, one may consider an array such as illustrated in Fig. 1 of the Discussion, but for which cooling of the solid is restricted to the regions immediately surrounding the tips of the shorter cracks, with the longer cracks being heated at their tips. In this instance, the shorter (cooled) cracks would propagate and the longer (heated) cracks would not. Many other such counterexamples can be identified; clearly the stability arguments of the Discussion require very special conditions (e.g. one-dimensional temperature distributions) if they are to hold.

In particular, the conclusions reached in the Discussion do not apply to the arrays of self-driven cracks studied by the authors[1]. There is an essential and important difference between the self-driven cracks and the crack arrays cited in the above Discussion. The analyses of these latter cracks (although they are intended to include the effects of convective fluid flow), consider only one-dimensional temperature fields. The self-driven cracks, on the other hand, experience a temperature distribution which is strongly two-dimensional, with regions of cooling being localized, to varying degrees, near the crack surfaces. While a formal stability analysis of the self-driven crack array has not been carried out, it is clear that the two-dimensionality of the temperature distribution is of fundamental importance, and the results of stability analyses, for cases in which the temperature field is one-dimensional, are not necessarily relevant. In fact, there is good reason to believe that such an array of self-driven cracks would propagate stably (i.e. with all cracks moving at the same speed), as will now be shown.

Consider an infinite array of identical, parallel cracks propagating with a common velocity v away from the surface of a half space (Fig. 1). The crack spacing s is much less than the crack lengths, the half space is initially at a uniform temperature T_0, and a